Causal Inference under Threshold Manipulation: A Bayesian Mixture Approach

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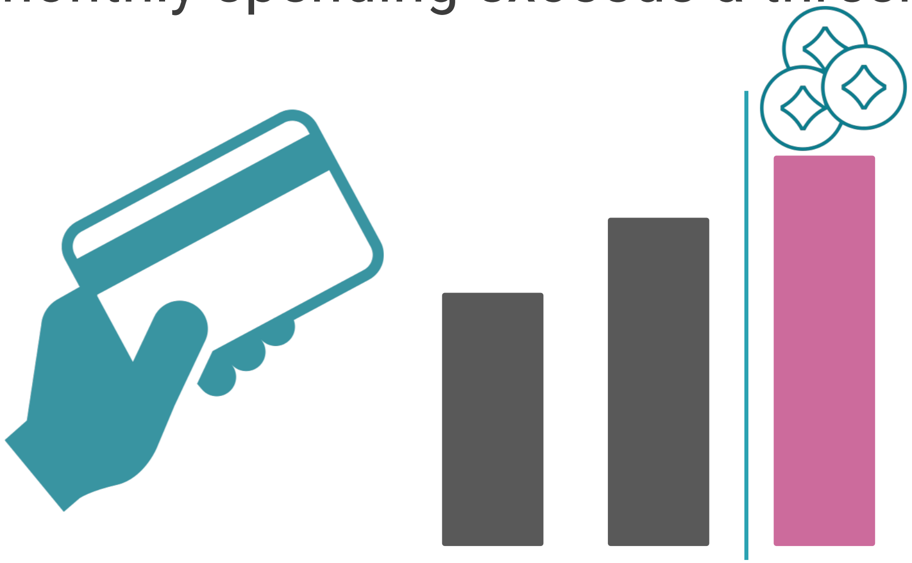
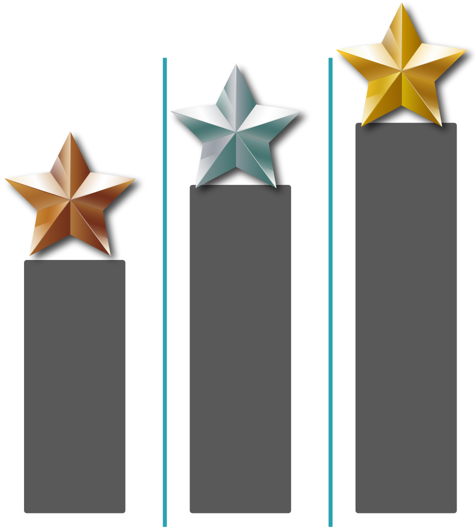
Use case: Threshold-based marketing strategies

Many marketing applications use **threshold** to offer rewards

Credit Card Incentive Programs:
Offering bonus points when a customer's monthly spending exceeds a threshold

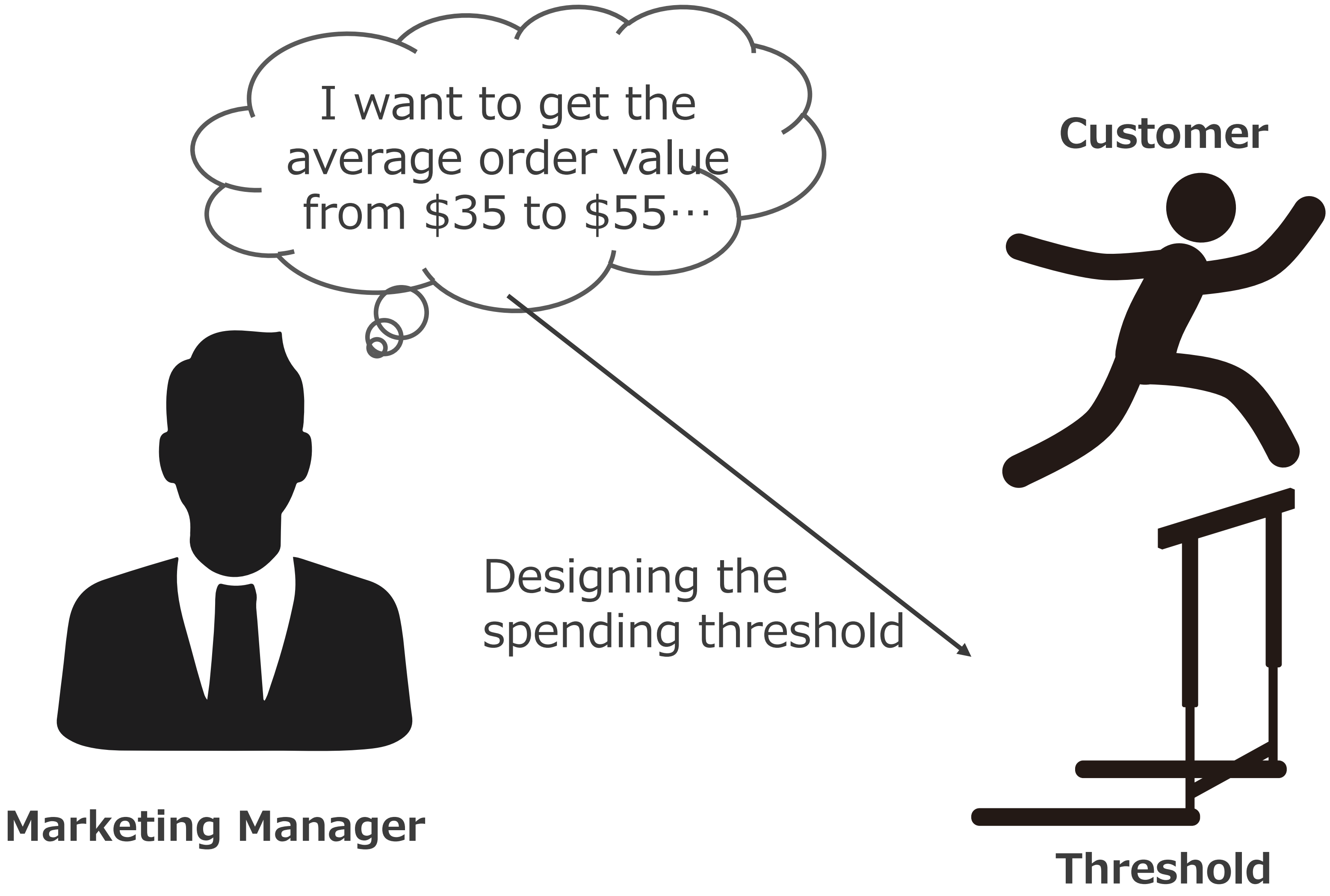


Loyalty Programs:
Upgrading a customer's status for surpassing an annual spending target



Motivation of estimating causal effects of thresholds

Estimating causal effects of thresholds is important for effective marketing

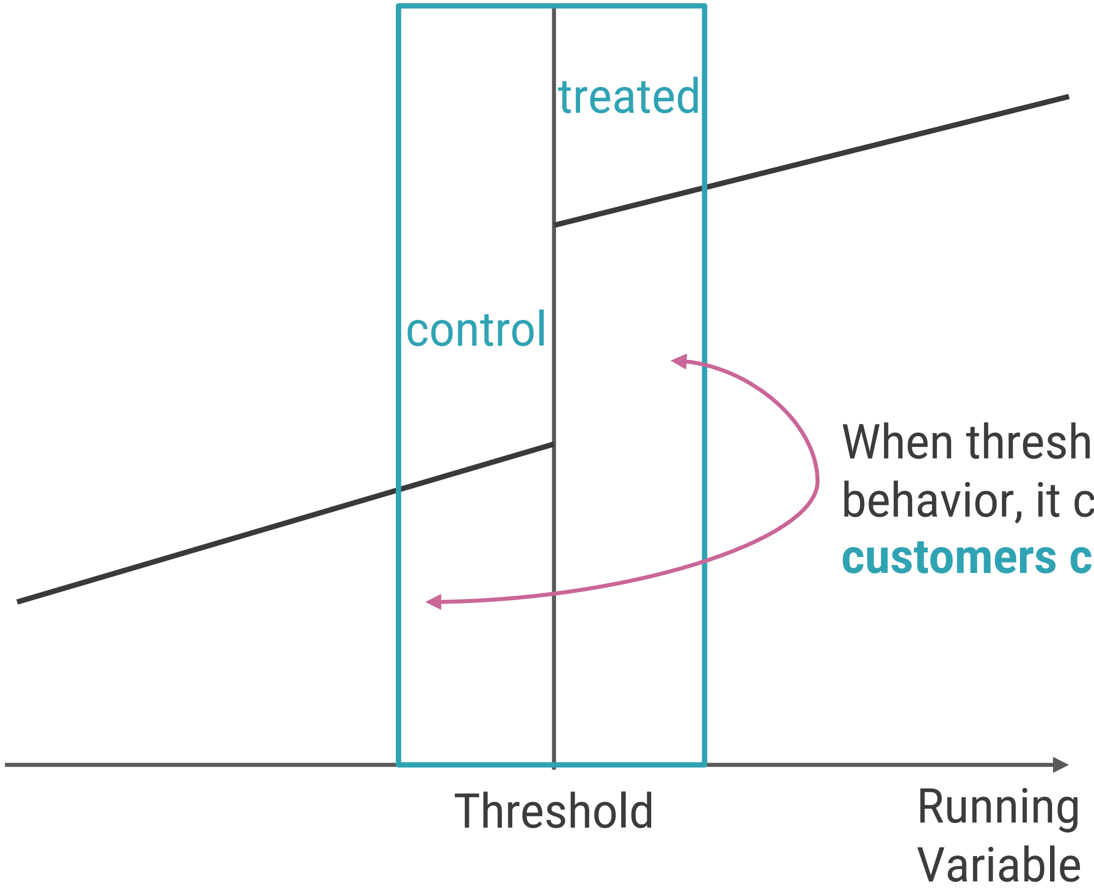


Motivation:
What is the Impact of Spending Thresholds?

Limitations of Regression Discontinuity Design (RDD)

The assumption of RDD can be violated
when customers strategically manipulate their behavior

Local randomization of RDD



When thresholds encourage strategic behavior, it creates **self-selection bias as customers can choose to cross the threshold**.

Basic setup

- Use the **potential outcome framework**:
  $Y(1)(\text{with threshold}), Y(0)(\text{without threshold})$

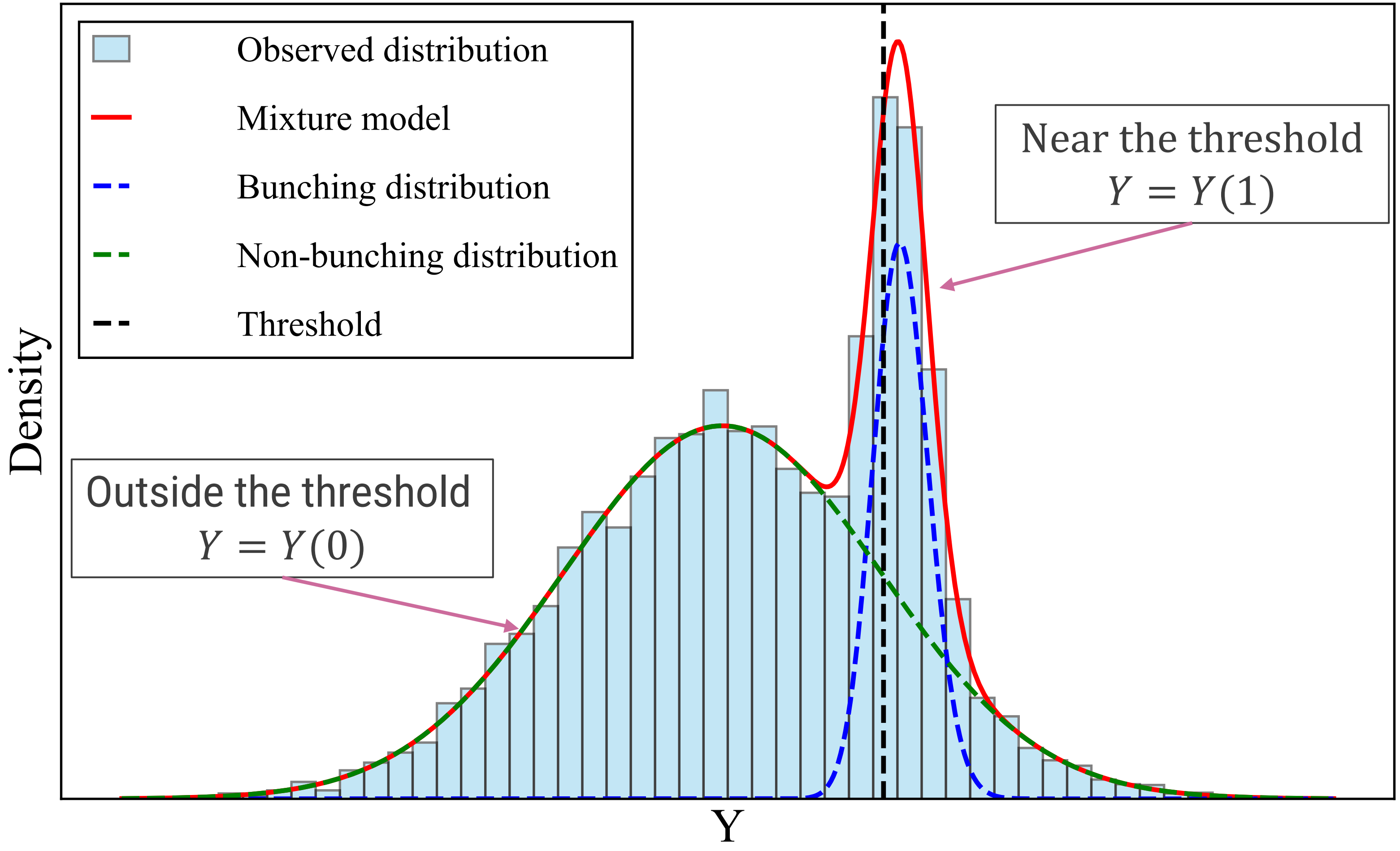
- Assume that all customers can be classified into two types:
  - **Bunching customers**: customers who strategically adjust their spending **near the threshold** to ensure they exceed it.
  - **Non-bunching customers**: customers whose spending behavior lies **outside the neighborhood of the threshold** and is therefore unaffected by it.

- The **average treatment effect of the threshold on customers** is defined as

$$\tau := \mathbb{E}[Y(1) - Y(0) \mid \underbrace{\underline{Y}^I \le Y(t) \le \overline{Y}^I}_{\text{Near the threshold}}, t \in \{0, 1\}]$$

Mixture model for bunching and non-bunching distributions

$$\underbrace{Y_i}_{\text{Observed data}} \sim \pi \underbrace{f(Y_i \mid \gamma)}_{\text{bunching customers}} + (1-\pi)\underbrace{g(Y_i \mid \theta)}_{\text{non-bunching customers}}$$



Bunching distribution f:
High density localized around the threshold

Non-bunching distribution g:
Widespread density across the entire data range

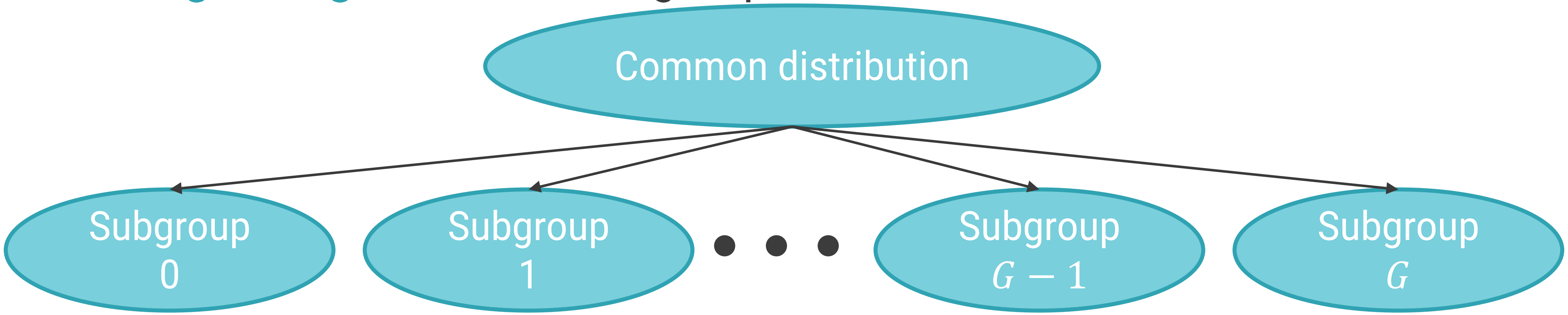
$$\tau = \frac{\int_{\underline{Y}^I \le Y(t) \le \overline{Y}^I} y f(y \mid \gamma) dy}{\int_{\underline{Y}^I \le Y(t) \le \overline{Y}^I} f(y \mid \gamma) dy} - \frac{\int_{\underline{Y}^I \le Y(t) \le \overline{Y}^I} y g(y \mid \theta) dy}{\int_{\underline{Y}^I \le Y(t) \le \overline{Y}^I} g(y \mid \theta) dy}$$

Causal effect estimation: a Bayesian approach

- We propose a Bayesian approach to fit our mixture model to the data (**B**ayesian **M**odeling of **T**hreshold **M**anipulation via **M**ixtures (**BMTM**))

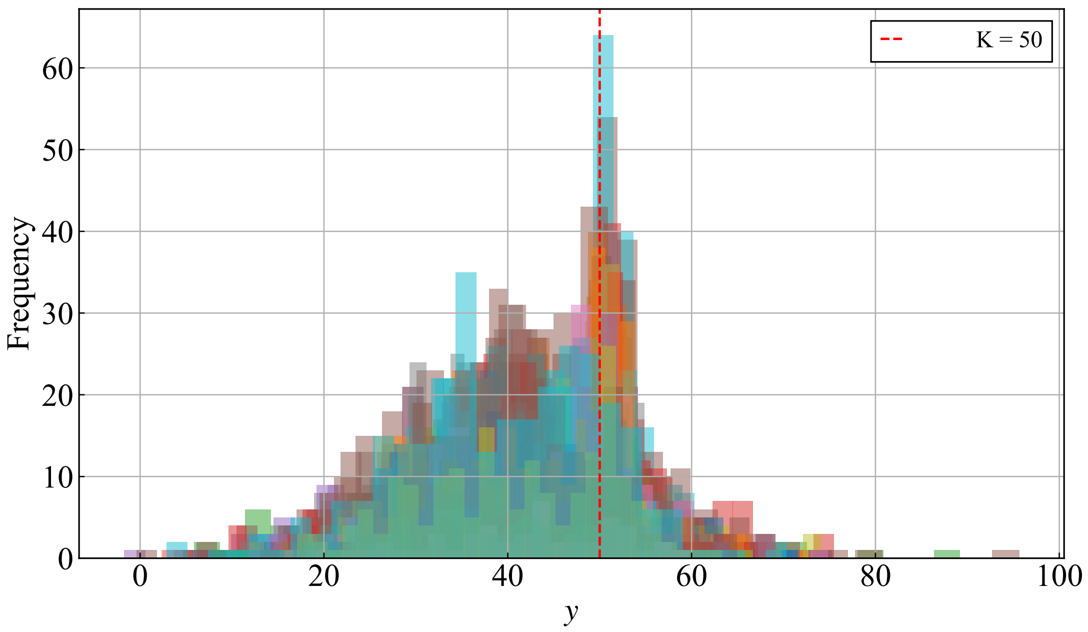
- This Bayesian approach provides three advantages:
  - **Incorporate prior knowledge** (e.g., bunching distribution is concentrated around K)
  - **Quantify uncertainty** of causal effects
  - **Easily extendable to hierarchical models** to estimate heterogeneous treatment effect (HTE)

- **Hierarchical Extension (HBMTM) for HTE provides stable estimate of each subgroup** by **"borrowing strength"** from other groups



Experimental results

Simulation results



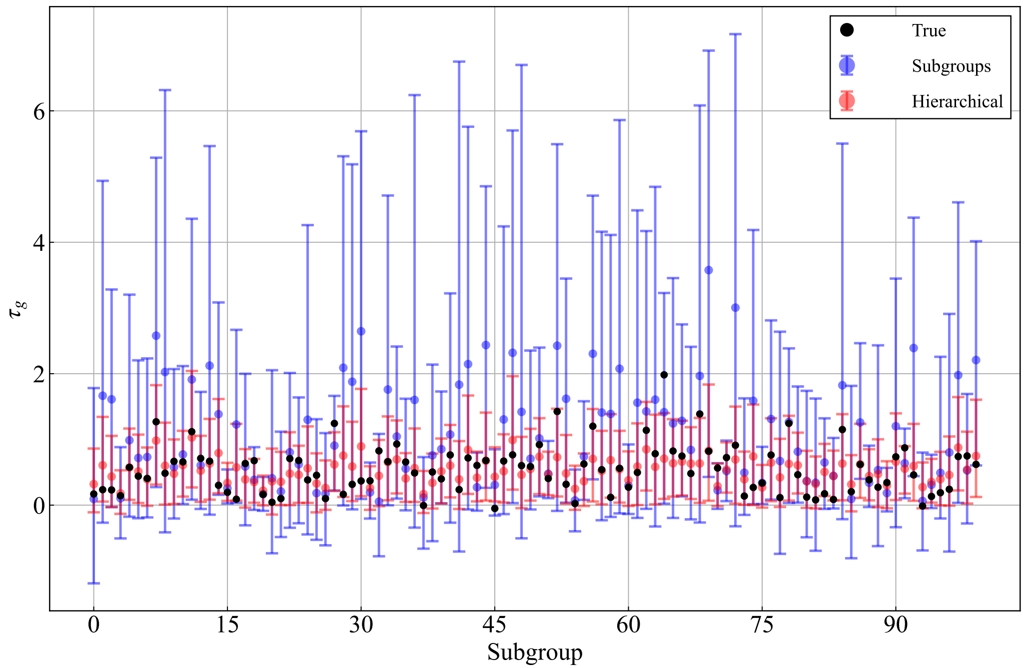
Simulation results

Method	MSE	IS
RDD	6.77	--
BMTM	0.68	3.42
HBMTM	**0.19**	**1.18**

MSE: Evaluates the accuracy of point estimates

IS: Evaluates the quality of interval estimates

Comparison of BMTM and HBMTM



Overall summary

- **Problem:** **Estimating the causal effect of marketing thresholds is crucial**, but **existing methods like RDD are unreliable when customers strategically manipulate their spending**

- **Core idea:** We propose **a new approach that models the observed spending distribution as a mixture of two populations**: customers who are strategically affected by the threshold, and those who not (**B**ayesian **M**odeling of **T**hreshold **M**anipulation via **M**ixtures (BMTM))

- **Extension:** We further extend BMTM into **a hierarchical model (HBMTM) to estimate heterogeneous treatment effects** across various customer subgroups.

- **Result:** Our simulation experiments demonstrated that **our proposed methods estimate the causal effect with far greater accuracy than conventional RDD**

More about the paper

- **Contact**: kousuke.kubota.xt@nttdocomo.com
- **arXiv**: coming soon! (an extended version)

Thank you!